

Straight line graphs

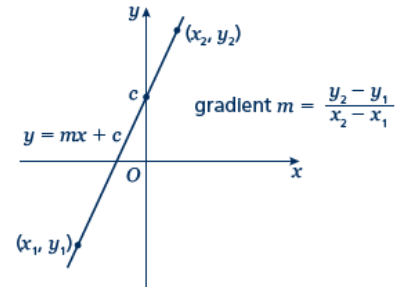
A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y -intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a , b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y -intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2} \text{ and } c = 3$ $\text{So } y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	<ol style="list-style-type: none"> 1 A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. 2 Rearrange the equation so all the terms are on one side and 0 is on the other side. 3 Multiply both sides by 2 to eliminate the denominator.
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Example 2 Find the gradient and the y -intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ $\text{Gradient} = m = \frac{2}{3}$ $y\text{-intercept} = c = -\frac{4}{3}$	<ol style="list-style-type: none"> 1 Make y the subject of the equation. 2 Divide all the terms by three to get the equation in the form $y = \dots$ 3 In the form $y = mx + c$, the gradient is m and the y-intercept is c.
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Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	<ol style="list-style-type: none"> 1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. 2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation. 3 Simplify and solve the equation. 4 Substitute $c = -2$ into the equation $y = 3x + c$
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Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	<ol style="list-style-type: none"> 1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. 2 Substitute the gradient into the equation of a straight line $y = mx + c$. 3 Substitute the coordinates of either point into the equation. 4 Simplify and solve the equation. 5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$
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Practice

1 Find the gradient and the y -intercept of the following equations.

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|---|---|
| <p>a $y = 3x + 5$</p> <p>c $2y = 4x - 3$</p> <p>e $2x - 3y - 7 = 0$</p> | <p>b $y = -\frac{1}{2}x - 7$</p> <p>d $x + y = 5$</p> <p>f $5x + y - 4 = 0$</p> |
|---|---|

Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	

- 3 Find, in the form $ax + by + c = 0$ where a , b and c are integers, an equation for each of the lines with the following gradients and y -intercepts.
- a gradient $-\frac{1}{2}$, y -intercept -7 b gradient 2 , y -intercept 0
- c gradient $\frac{2}{3}$, y -intercept 4 d gradient -1.2 , y -intercept -2
- 4 Write an equation for the line which passes through the point $(2, 5)$ and has gradient 4 .
- 5 Write an equation for the line which passes through the point $(6, 3)$ and has gradient $-\frac{2}{3}$
- 6 Write an equation for the line passing through each of the following pairs of points.
- a $(4, 5)$, $(10, 17)$ b $(0, 6)$, $(-4, 8)$
- c $(-1, -7)$, $(5, 23)$ d $(3, 10)$, $(4, 7)$

Extend

- 7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

- 1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$
c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$
e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

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Gradient	y-intercept	Equation of the line
5	0	$y = 5x$
-3	2	$y = -3x + 2$
4	-7	$y = 4x - 7$

- 3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$
c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or (4, -3).