

# Solving simultaneous equations graphically

## A LEVEL LINKS

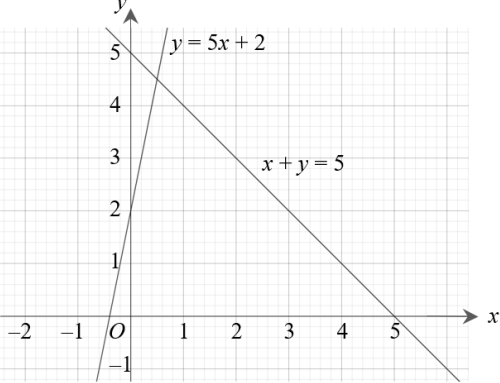
Scheme of work: 1c. Equations – quadratic/linear simultaneous

## Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

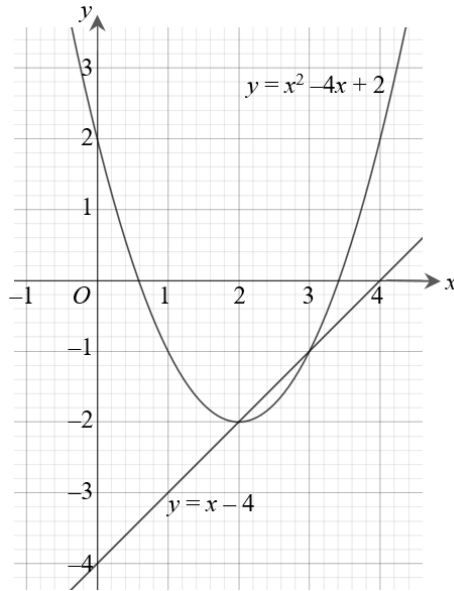
## Examples

**Example 1** Solve the simultaneous equations  $y = 5x + 2$  and  $x + y = 5$  graphically.

<p><math>y = 5 - x</math></p> <p><math>y = 5 - x</math> has gradient <math>-1</math> and <math>y</math>-intercept <math>5</math>.  <math>y = 5x + 2</math> has gradient <math>5</math> and <math>y</math>-intercept <math>2</math>.</p>  <p>Lines intersect at  <math>x = 0.5, y = 4.5</math></p> <p>Check:          First equation <math>y = 5x + 2</math>:  <math>4.5 = 5 \times 0.5 + 2</math>      YES          Second equation <math>x + y = 5</math>:  <math>0.5 + 4.5 = 5</math>      YES</p>	<ol style="list-style-type: none"> <li><b>1</b> Rearrange the equation <math>x + y = 5</math> to make <math>y</math> the subject.</li> <li><b>2</b> Plot both graphs on the same grid using the gradients and <math>y</math>-intercepts.</li> <li><b>3</b> The solutions of the simultaneous equations are the point of intersection.</li> <li><b>4</b> Check your solutions by substituting the values into both equations.</li> </ol>
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**Example 2** Solve the simultaneous equations  $y = x - 4$  and  $y = x^2 - 4x + 2$  graphically.

<b>x</b>	0	1	2	3	4
<b>y</b>	2	-1	-2	-1	2



The line and curve intersect at  
 $x = 3, y = -1$  and  $x = 2, y = -2$

Check:

First equation  $y = x - 4$ :

$$-1 = 3 - 4 \quad \text{YES}$$

$$-2 = 2 - 4 \quad \text{YES}$$

Second equation  $y = x^2 - 4x + 2$ :

$$-1 = 3^2 - 4 \times 3 + 2 \quad \text{YES}$$

$$-2 = 2^2 - 4 \times 2 + 2 \quad \text{YES}$$

**1** Construct a table of values and calculate the points for the quadratic equation.

**2** Plot the graph.

**3** Plot the linear graph on the same grid using the gradient and  $y$ -intercept.  
 $y = x - 4$  has gradient 1 and  $y$ -intercept  $-4$ .

**4** The solutions of the simultaneous equations are the points of intersection.

**5** Check your solutions by substituting the values into both equations.

## Practice

**1** Solve these pairs of simultaneous equations graphically.

**a**  $y = 3x - 1$  and  $y = x + 3$

**b**  $y = x - 5$  and  $y = 7 - 5x$

**c**  $y = 3x + 4$  and  $y = 2 - x$

**2** Solve these pairs of simultaneous equations graphically.

**a**  $x + y = 0$  and  $y = 2x + 6$

**b**  $4x + 2y = 3$  and  $y = 3x - 1$

**c**  $2x + y + 4 = 0$  and  $2y = 3x - 1$

### Hint

Rearrange the equation to make  $y$  the subject.

- 3 Solve these pairs of simultaneous equations graphically.
- a  $y = x - 1$  and  $y = x^2 - 4x + 3$
  - b  $y = 1 - 3x$  and  $y = x^2 - 3x - 3$
  - c  $y = 3 - x$  and  $y = x^2 + 2x + 5$
- 4 Solve the simultaneous equations  $x + y = 1$  and  $x^2 + y^2 = 25$  graphically.

### Extend

- 5 a Solve the simultaneous equations  $2x + y = 3$  and  $x^2 + y = 4$
- i graphically
  - ii algebraically to 2 decimal places.
- b Which method gives the more accurate solutions? Explain your answer.

## Answers

- 1**
- a**  $x = 2, y = 5$
  - b**  $x = 2, y = -3$
  - c**  $x = -0.5, y = 2.5$
- 2**
- a**  $x = -2, y = 2$
  - b**  $x = 0.5, y = 0.5$
  - c**  $x = -1, y = -2$
- 3**
- a**  $x = 1, y = 0$  and  $x = 4, y = 3$
  - b**  $x = -2, y = 7$  and  $x = 2, y = -5$
  - c**  $x = -2, y = 5$  and  $x = -1, y = 4$
- 4**  $x = -3, y = 4$  and  $x = 4, y = -3$
- 5**
- a**
    - i**  $x = 2.5, y = -2$  and  $x = -0.5, y = 4$
    - ii**  $x = 2.41, y = -1.83$  and  $x = -0.41, y = 3.83$
  - b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.