

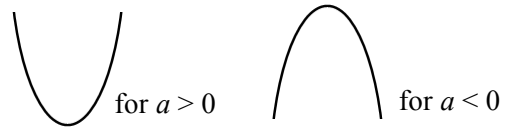
# Sketching quadratic graphs

## A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- The graph of the quadratic function  $y = ax^2 + bx + c$ , where  $a \neq 0$ , is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the  $y$ -axis substitute  $x = 0$  into the function.
- To find where the curve intersects the  $x$ -axis substitute  $y = 0$  into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.



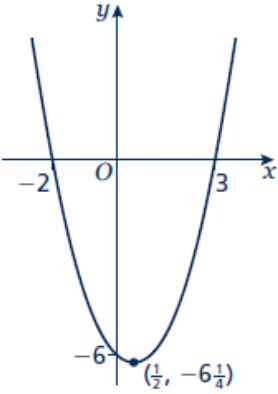
## Examples

**Example 1** Sketch the graph of  $y = x^2$ .

	<p>The graph of <math>y = x^2</math> is a parabola.</p> <p>When <math>x = 0</math>, <math>y = 0</math>.</p> <p><math>a = 1</math> which is greater than zero, so the graph has the shape:</p>
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**Example 2** Sketch the graph of  $y = x^2 - x - 6$ .

<p>When <math>x = 0</math>, <math>y = 0^2 - 0 - 6 = -6</math> So the graph intersects the <math>y</math>-axis at <math>(0, -6)</math> When <math>y = 0</math>, <math>x^2 - x - 6 = 0</math> <math>(x + 2)(x - 3) = 0</math> <math>x = -2</math> or <math>x = 3</math></p> <p>So, the graph intersects the <math>x</math>-axis at <math>(-2, 0)</math> and <math>(3, 0)</math></p>	<ol style="list-style-type: none"> <li>Find where the graph intersects the <math>y</math>-axis by substituting <math>x = 0</math>.</li> <li>Find where the graph intersects the <math>x</math>-axis by substituting <math>y = 0</math>.</li> <li>Solve the equation by factorising.</li> <li>Solve <math>(x + 2) = 0</math> and <math>(x - 3) = 0</math>.</li> <li><math>a = 1</math> which is greater than zero, so the graph has the shape:</li> </ol> <p>(continued on next page)</p> <ol style="list-style-type: none"> <li>To find the turning point, complete</li> </ol>
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$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$ <p>When <math>\left(x - \frac{1}{2}\right)^2 = 0</math>, <math>x = \frac{1}{2}</math> and <math>y = -\frac{25}{4}</math>, so the turning point is at the point <math>\left(\frac{1}{2}, -\frac{25}{4}\right)</math></p> 	<p>the square.</p> <p>7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.</p>
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## Practice

- Sketch the graph of  $y = -x^2$ .
- Sketch each graph, labelling where the curve crosses the axes.
 

<b>a</b> $y = (x + 2)(x - 1)$	<b>b</b> $y = x(x - 3)$	<b>c</b> $y = (x + 1)(x + 5)$
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- Sketch each graph, labelling where the curve crosses the axes.
 

<b>a</b> $y = x^2 - x - 6$	<b>b</b> $y = x^2 - 5x + 4$	<b>c</b> $y = x^2 - 4$
<b>d</b> $y = x^2 + 4x$	<b>e</b> $y = 9 - x^2$	<b>f</b> $y = x^2 + 2x - 3$
- Sketch the graph of  $y = 2x^2 + 5x - 3$ , labelling where the curve crosses the axes.

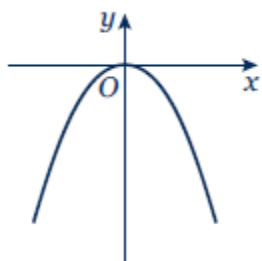
## Extend

- Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.
 

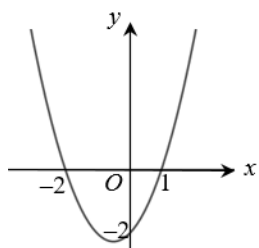
<b>a</b> $y = x^2 - 5x + 6$	<b>b</b> $y = -x^2 + 7x - 12$	<b>c</b> $y = -x^2 + 4x$
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- Sketch the graph of  $y = x^2 + 2x + 1$ . Label where the curve crosses the axes and write down the equation of the line of symmetry.

## Answers

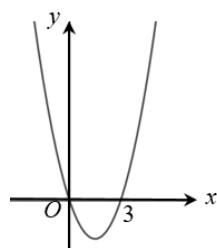
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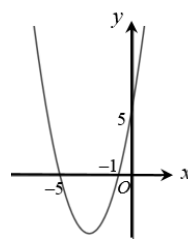
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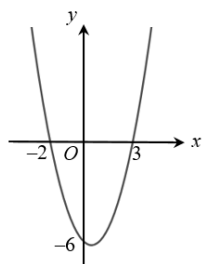
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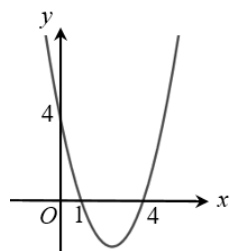
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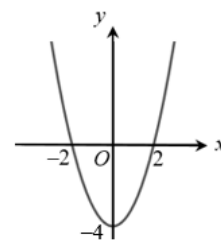
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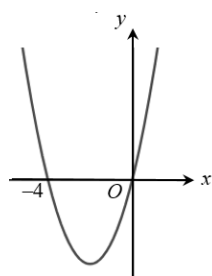
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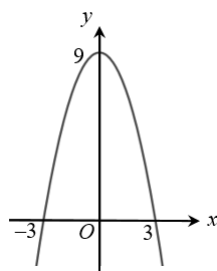
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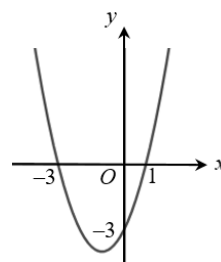
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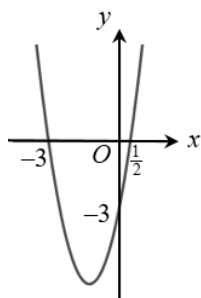
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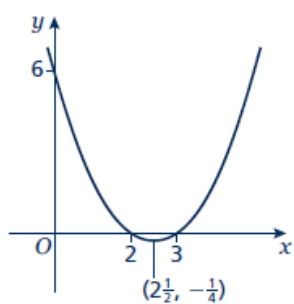
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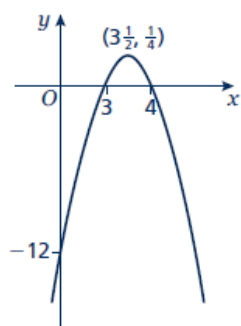
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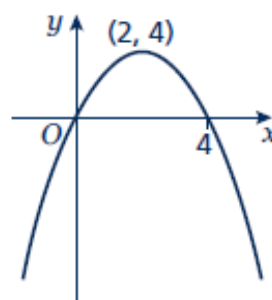
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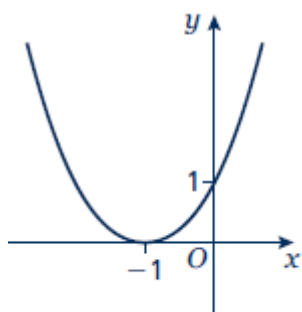
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c



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Line of symmetry at  $x = -1$ .