

# Completing the square

## A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Completing the square for a quadratic rearranges  $ax^2 + bx + c$  into the form  $p(x + q)^2 + r$
- If  $a \neq 1$ , then factorise using  $a$  as a common factor.

## Examples

**Example 1** Complete the square for the quadratic expression  $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	<p><b>1</b> Write <math>x^2 + bx + c</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c</math></p> <p><b>2</b> Simplify</p>
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**Example 2** Write  $2x^2 - 5x + 1$  in the form  $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	<p><b>1</b> Before completing the square write <math>ax^2 + bx + c</math> in the form <math>a\left(x^2 + \frac{b}{a}x\right) + c</math></p> <p><b>2</b> Now complete the square by writing <math>x^2 - \frac{5}{2}x</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2</math></p> <p><b>3</b> Expand the square brackets – don't forget to multiply <math>\left(\frac{5}{4}\right)^2</math> by the factor of 2</p> <p><b>4</b> Simplify</p>
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## Practice

- 1 Write the following quadratic expressions in the form  $(x + p)^2 + q$
- |                         |                          |
|-------------------------|--------------------------|
| <b>a</b> $x^2 + 4x + 3$ | <b>b</b> $x^2 - 10x - 3$ |
| <b>c</b> $x^2 - 8x$     | <b>d</b> $x^2 + 6x$      |
| <b>e</b> $x^2 - 2x + 7$ | <b>f</b> $x^2 + 3x - 2$  |
- 2 Write the following quadratic expressions in the form  $p(x + q)^2 + r$
- |                           |                           |
|---------------------------|---------------------------|
| <b>a</b> $2x^2 - 8x - 16$ | <b>b</b> $4x^2 - 8x - 16$ |
| <b>c</b> $3x^2 + 12x - 9$ | <b>d</b> $2x^2 + 6x - 8$  |
- 3 Complete the square.
- |                          |                          |
|--------------------------|--------------------------|
| <b>a</b> $2x^2 + 3x + 6$ | <b>b</b> $3x^2 - 2x$     |
| <b>c</b> $5x^2 + 3x$     | <b>d</b> $3x^2 + 5x + 3$ |

## Extend

- 4 Write  $(25x^2 + 30x + 12)$  in the form  $(ax + b)^2 + c$ .

**Answers**

**1 a**  $(x + 2)^2 - 1$

**b**  $(x - 5)^2 - 28$

**c**  $(x - 4)^2 - 16$

**d**  $(x + 3)^2 - 9$

**e**  $(x - 1)^2 + 6$

**f**  $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

**2 a**  $2(x - 2)^2 - 24$

**b**  $4(x - 1)^2 - 20$

**c**  $3(x + 2)^2 - 21$

**d**  $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

**3 a**  $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$

**b**  $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

**c**  $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$

**d**  $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

**4**  $(5x + 3)^2 + 3$